[13.18] Let *V* be an n-dimensional vector space. The linear transformation *T* : *V* → *V* is nonsingular ⇒ T has an inverse

Proof: Let  be a basis for *V*. Let *bk* = *T ak*. Then { *bk* } is also a basis for *V* :

Suppose (Einstein Summation Convention.) Then  Thus { *bk* } is a collection of linearly independent vectors of size *n*, and hence { *bk* } is a basis for *V*.

Define  We show  by showing  (This is equivalent to 

Let  Then  ✔

Since  is a basis,  So  ✔

Note: Penrose said not to use explicit expressions. By that, he may have meant to do a coordinate-free construction, which I didn’t because I assumed a basis { *ak* }. Beckmann avoided using a basis but otherwise his proof seems as “explicit” as mine. He proved that nonsingular ⇒ 1-1 and onto. Therefore  He defined  Thus  ✔

Next, he fixed *v* so that there is a *w* such that  So  Since  and *T* is 1-1, then  i.e.,   ✔

Therefore he concluded that 